

## On BPP versus $\text{NP} \cup \text{coNP}$ for ordered read-once branching programs

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### Abstract

We investigate the relationship between probabilistic and nondeterministic complexity classes PP, BPP, NP and coNP with respect to ordered read-once branching programs (OBDDs). We exhibit two explicit Boolean functions  $q_n, R_n$  such that: (1)  $q_n : \{0, 1\}^n \rightarrow \{0, 1\}$  belongs to  $\text{BPP} \setminus (\text{NP} \cup \text{coNP})$  in the context of OBDDs; (2)  $R_n : \{0, 1\}^n \rightarrow \{0, 1\}$  belongs to  $\text{PP} \setminus (\text{BPP} \cup \text{NP} \cup \text{coNP})$  in the context of OBDDs. Both of these functions are not in  $\text{AC}^0$ . © 2001 Elsevier Science B.V. All rights reserved.

### 1. Preliminaries

Ordered binary decision diagrams (for short OBDDs) are also known as deterministic ordered (or oblivious) read-once branching programs. OBDDs are important tools in the field of digital design and hardware verification (see, for example, [8, 21]). The reason for this is that the manipulation (testing for equivalence and other Boolean operations) with OBDDs can be performed in deterministic polynomial time. But for this convenience we “pay a sensible tax”: some important (for practice) Boolean functions cannot be represented by polynomial size OBDDs (see, for example, [16]). So, the important task is to investigate reasonable generalizations of OBDD model of

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